An Analytical Solution for the Coupled Stripline-Like Microstrip Line Problem

DOREL HOMENTCOVSCHI, ANTON MANOLESCU, ANCA MANUELA MANOLESCU, AND LIVIU KREINDLER

Abstract —An analytical method for determining the Maxwell's capacitance matrix of multiconductor coupled stripline-like microstrip lines in an inhomogeneous medium is presented. The method is based on conformal mapping and the theory of singular integral equations.

I. Introduction

RECENTLY the study of multiconductor coupled stripline-like microstrip lines has attracted considerable interest due to the favorable properties of these lines for developing new microwave integrated circuits such as directional couplers and parallel coupled filters. In contrast to the so-called microstrip line classical half-shielded structure, the full-shielded structure referred to as stripline-like microstrip [1] offers the advantage of having the mode velocities independent of the strip widths and spacing, and as a consequence a very good directivity and well-defined electrical behavior.

For multiconductor coupled structures with very tight coupling (interdigitated directional couplers, high-order parallel coupled line filters) an accurate calculation method is required in order to estimate the influence of all the conducting strips. A number of methods to perform this analysis are currently used; all of them solve the Laplace equation for the bidimensional electrostatic equivalent problem for low-order, quasi-TEM modes. One can mention several commonly used methods, such as conformal mapping for simple symmetrical cases [2], numerical methods based on lattice approximations [3], variational methods [4], and methods based on solving the integral equations derived from Green's functions [5]–[7]. Although some of the numerical methods are quite general, none of the above can be considered suitable for analysis and especially for synthesis of all the high-performance structures.

The present paper presents a new analytical method for determining Maxwell's capacitance matrix for multiconductor stripline-like microstrips with coupled conductors with arbitrary widths and spacings and inhomogeneous media. It appears that the more general case of unsymmetrical configurations cannot be solved by a similar method.

Manuscript received August 19, 1987; revised January 14, 1988.
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IEEE Log Number 8821074.

II. Basic Configuration

The multiconductor system to be analyzed consists of n zero-thickness conducting strips A_kB_k, with arbitrary widths and spacings, located on a dielectric substrate of thickness h. The system is fully shielded by ground planes on all sides, as shown in Fig. 1, and is subject to the constraint that the shield spacing l equals the substrate thickness h. However the relative dielectric constants ϵ_1 and ϵ_2 corresponding to the upper and lower dielectric media may be different.

The electrostatic field $\boldsymbol{E}(E_x, E_y)$ in the two dielectric media inside the shielded box can be expressed by means of electrostatic potentials as

$$E_x^{(j)}(x,y) = -\partial\psi^{(j)}/\partial x$$
$$E_y^{(j)}(x,y) = -\partial\psi^{(j)}/\partial y, \qquad j=1,2. \tag{1}$$

As $\psi^{(j)}(x,y)$ are harmonic functions, we can introduce the harmonic conjugate functions $\varphi^{(j)}(x,y)$—the field functions. Therefore, the complex potential functions

$$f^{(j)}(z) = \varphi^{(j)}(x,y) + i\cdot\psi^{(j)}(x,y), \qquad j=1,2 \tag{2}$$

are holomorphic in the complex variable $z = x + i\cdot y$ inside the two dielectric media.

On the shielded box the potential functions must vanish, i.e.,

$$\psi^{(1)}(x,y) = 0 \qquad \text{for } y>0$$
$$\psi^{(2)}(x,y) = 0 \qquad \text{for } y<0, \quad \text{on the box.} \tag{3}$$

On the other hand on the symmetry axis we must have the physical conditions

$$D_y(x,+0) - D_y(x,-0) = \rho(x) \tag{4a}$$

$$E_x(x,+0) - E_x(x,-0) = 0 \tag{4b}$$

where $\rho(x)$ is the surface density of the electrical charges and $\boldsymbol{D}(D_x, D_y)$ is the electrical induction.

On the insulating segments B_kA_{k+1} we must have $\rho(x) = 0$; by using the field function the relation (4a) gives

$$-\epsilon_1\cdot\varphi^{(1)}(x,0) + \epsilon_2\cdot\varphi^{(2)}(x,0) = -q_k,$$
$$x\in B_kA_{k+1}, \quad k=1,\cdots,n \tag{5}$$

where q_k are unknown constants. Relation (4b) gives also

$$\psi^{(1)}(x,0) = \psi^{(2)}(x,0). \tag{6}$$

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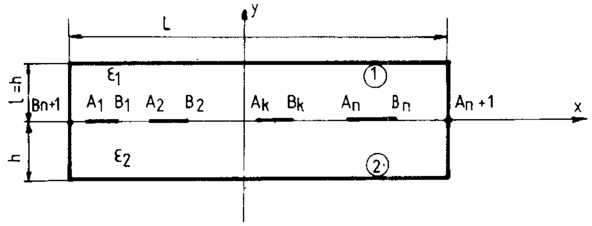

Fig. 1. Full-shielded multiconductor coupled striplines.

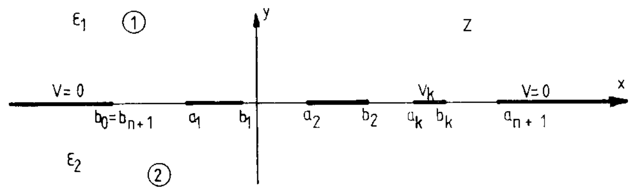

Fig. 2. The canonic domain obtained by conformal mapping of the domain in Fig. 1.

On the conducting strip A_kB_k we must have

$$\psi^{(1)}(x,0)=\psi^{(2)}(x,0)=V_k,\qquad k=1,\cdots,n,\qquad x\in A_kB_k \tag{7}$$

where V_k is the potential of the kth electrode. Relation (4a) determines in this case the function $\rho(x)$. The total charge Q_k on the strip A_kB_k is given by

$$Q_k=\int_{A_k}^{B_k}\rho(x)\cdot dx=-q_{k+1}+q_k.$$

Hence

$$q_k=Q_k+q_{k+1}=\sum_{j=k}^{n}Q_j+q_{n+1}\qquad (k=1,\cdots,n). \tag{8}$$

Finally, relations (5) and (6) are the boundary conditions on the insulating lines and relations (7) are the boundary conditions on the conducting strips.

III. Determination of Maxwell's Capacitance Matrix

In order to solve the above settled boundary value problem, we conformally transform the domain filled by the two dielectric materials into a canonic domain. The upper region may be conformally mapped onto the upper complex half-plane $\mathrm{Im}\{Z\}>0$. (If the studied domain is actually a rectangular one, as in Fig. 1, the mapping function $z=w^{-1}(Z)$ is given by the Schwarz–Christoffel formula and will be expressed by incomplete elliptical functions of the first kind.) Let $(-\infty,b_0)\cup(a_{n+1},\infty)$ be the image of the upper side of the shielded box and the segment (a_k,b_k) the image of the strip A_kB_k ($k=1,2,\cdots,n$). By symmetry reasons the lower region of the domain in Fig. 1 will be conformally mapped on the lower half-plane (Fig. 2).

Thus the method can be used for more general geometries of the two dielectric media. The only restriction is the symmetry of the two dielectric domains with respect to the electrode line. In the sequel the actual shape of the shielded boundary is involved only through the abscissas a_k, b_k on the real axis $Y=0$ in the Z plane, corresponding to the points A_k, B_k.

By conformal mapping the complex potentials $f^{(1)}(z)$, $f^{(2)}(z)$ become two holomorphic functions in, respectively, the upper and lower half-planes $F^{(1)}(Z)$, $F^{(2)}(Z)$.

We can write

$$F^{(j)}(Z)=-\frac{(-1)^j}{\pi}\int_{-\infty}^{+\infty}\frac{\mu(t)}{t-Z}\cdot dt\qquad (j=1,2) \tag{9}$$

where the real function $\mu(t)$ must be determined by taking into account the boundary conditions:

$$\Psi^{(1)}(X,0)=\Psi^{(2)}(X,0)=\begin{cases}0 & \text{for } X(-\infty,b_0)\cup(a_{n+1},\infty)\\ V_k & \text{for } X\in(a_k,b_k),\ (k=1,\cdots,n)\end{cases} \tag{10}$$

and

$$\begin{aligned}&\epsilon_1\cdot\Phi^{(1)}(X,0)-\epsilon_2\cdot\Phi^{(2)}(X,0)=q_k && \text{for } X\in(b_k,a_{k+1})\\ &\Psi^{(1)}(X,0)=\Psi^{(2)}(X,0), && k=0,1,\cdots,n.\end{aligned} \tag{11}$$

The values of functions $F^{(1)}(Z)$ and $F^{(2)}(Z)$ on the real axis can be obtained by using the Plemelj relations [8]:

$$\begin{aligned}F^{(j)}(X)&=\Phi^{(j)}(X,0)+i\Psi^{(j)}(X,0)\\&=i\mu(X)-\frac{(-1)^j}{\pi}\int_{-\infty}^{\prime+\infty}\frac{\mu(t)}{t-X}dt\qquad (j=1,2)\end{aligned} \tag{12}$$

where $\int'$ stands for the Cauchy principal value of the integral.

Relations (9)–(11) give

$$\begin{aligned}&\mu(X)=0 && \text{for } X\in(-\infty,b_0)\cup(a_{n+1},\infty)\\ &\mu(X)=V_k && \text{for } X\in(a_k,b_k),\qquad k=1,\cdots,n\\ &\frac{\epsilon_1+\epsilon_2}{\pi}\int_{-\infty}^{+\infty}\frac{\mu(t)}{t-X}dt=q_k\\ & && \text{for } X\in(b_k,a_{k+1}),\ k=0,1,\cdots,n.\end{aligned} \tag{13}$$

The first two relations (13) determine the values of the function $\mu(X)$ on the electrodes; the last relation (13) gives the equation of the problem

$$\begin{aligned}\sum_{j=0}^{n}\frac{1}{\pi}\int_{b_j}^{\prime a_{j+1}}\frac{\mu(t)}{t-X}dt&=\frac{q_k}{\epsilon_1+\epsilon_2}-\sum_{j=1}^{n}\frac{1}{\pi}\int_{a_j}^{b_j}\frac{V_j}{t-X}dt\\&\equiv f(X),\qquad X\in(b_k,a_{k+1}),\ k=0,1,\cdots,n.\end{aligned} \tag{14}$$

In order to solve the singular integral equation (14) let us consider the complex variable function

$$F(Z)=\Phi(X,Y)+i\Psi(X,Y)=\sum_{j=0}^{n}\frac{1}{i\pi}\int_{b_j}^{a_{j+1}}\frac{\mu(t)}{t-Z}dt.$$

This is a holomorphic function in the upper half-plane. By using the Plemelj relations we get

$$\Phi(X,0)=0 \qquad \text{for } X\in(-\infty,b_0)\cup(a_k,b_k)\cup(a_{n+1},\infty)$$
$$\Psi(X,0)=-f(X) \qquad \text{for } X\in(b_k,a_{k+1}),\ k=0,1,\cdots,n.$$

Thus, the function $F(Z)$ is the solution of a Volterra boundary value problem [8], [9], [15]. Finally, the existence of a bounded solution of the integral equation is conditioned by compatibility conditions [9]:

$$\sum_{j=0}^{n}\int_{b_j}^{a_{j+1}}\frac{f(t)t^{l-1}}{\sqrt{P(t)}}\,dt=0 \qquad (l=1,\cdots,n). \tag{15}$$

In the case of (14) these conditions become

$$\sum_{k=0}^{n}\frac{q_k}{\epsilon_1+\epsilon_2}\int_{b_k}^{a_{k+1}}\frac{t^{l-1}}{\sqrt{P(t)}}\,dt-\sum_{j=1}^{n}V_j\int_{a_j}^{b_j}dt'$$
$$\cdot\sum_{k=0}^{n}\frac{1}{\pi}\int_{b_k}^{'a_{k+1}}\frac{t^{l-1}}{(t'-t)\sqrt{P(t)}}\,dt=0, \qquad l=1,2,\cdots,n \tag{16}$$

where

$$P(Z)=\prod_{j=1}^{n+1}(Z-a_j)\cdot(Z-b_j).$$

If we substitute q_k given by relation (8) into formula (16), we obtain

$$\frac{1}{\epsilon_1+\epsilon_2}\sum_{k=1}^{n}Q_k\sum_{j=0}^{k-1}\int_{b_j}^{a_{j+1}}\frac{t^{l-1}}{\sqrt{P(t)}}\,dt$$
$$+i\sum_{k=1}^{n}V_k\int_{a_k}^{b_k}\frac{t^{l-1}}{\sqrt{P(t)}}\,dt=0. \tag{17}$$

Finally, the conditions for the existence of bounded solutions of (14) become

$$\sum_{k=1}^{n}N_{lk}\cdot Q_k=(\epsilon_1+\epsilon_2)\sum_{k=1}^{n}M_{lk}\cdot V_k, \qquad l=1,\cdots,n \tag{18}$$

where we use

$$A_{lk}=(-1)^k\int_{a_k}^{b_k}\frac{t^{l-1}}{\sqrt{|P(t)|}}\,dt$$
$$B_{lk}=(-1)^{k+1}\int_{b_k}^{a_{k+1}}\frac{t^{l-1}}{\sqrt{P(t)}}\,dt$$
$$M_{lk}=A_{lk};\quad N_{lk}=\sum_{j=0}^{k-1}B_{lj}, \qquad l=1,\cdots,n. \tag{19}$$

Relation (18) gives the Maxwell capacitance matrix $\boldsymbol{C}$:

$$\boldsymbol{C}=(\epsilon_1+\epsilon_2)\boldsymbol{N}^{-1}\cdot\boldsymbol{M} \tag{20}$$

where the matrices $\boldsymbol{M}, \boldsymbol{N}$ are defined by relation (19).

Let us denote now by $\boldsymbol{C}_{\text{hom}}$ the Maxwell capacitance matrix corresponding to the homogeneous dielectric medium ($\epsilon_1=\epsilon_2=\epsilon_0$). Relations (20) give

$$\boldsymbol{C}=\frac{\epsilon_1+\epsilon_2}{2\epsilon_0}\boldsymbol{C}_{\text{hom}}. \tag{20'}$$

Therefore, in order to obtain the Maxwell capacitance matrix of a multiconductor system in a stratified dielectric (as represented in Fig. 1), it is necessary to know only the Maxwell capacitance matrix $\boldsymbol{C}_{\text{hom}}$ of the same multiconductor structure in free space.

In this way we obtained an analytical solution for the Maxwell capacitance matrix of the considered structure in terms of hyperelliptic integrals depending on the structure geometry only by means of constants $a_1,\cdots,a_{n+1}$ and $b_1,\cdots,b_{n+1}$. These relations are similar to those characterizing the impedance matrix of a resistive distributed structure [9].

IV. Applications

Let us now apply the above method to some actual structures. We consider two kinds of problems, the first consisting of simple structures with one or two coupled striplines. In this case relations (20) give analytical formulas for the capacitances in terms of elliptical functions. The other type of application concerns the general case of multiconductor coupled structures for which simple analytical expressions are no longer available. Accordingly the determination of the capacitance matrix requires the use of the general formula (20).

In most applications the domains of interest are the rectangular box and the domain between two parallel ground planes. If the dielectric media fill the rectangle $B_{n+1}<x<A_{n+1}$, $-h<y<h$, the conformal mapping function is obtained by means of elliptical functions. The abscissas a_k, b_k of the points on the X axis corresponding to the electrode extremities are obtained in terms of Jacobi's sn function

$$X=\text{sn}(x\cdot K/L,k) \tag{21}$$

where the modulus k is the solution of the equation

$$K(k)/K(k')=L/h, \qquad k'=\sqrt{1-k^2}. \tag{22}$$

Here $K(k)$ is the complete elliptical integral of the first kind.

As the lateral sides of the rectangle are approaching infinity ($L\to\infty$) the domain will tend to the strip $-h<y<h$ and the abscissas a_k, b_k will be given by the formula

$$X=\tanh(\pi x/(2h)) \tag{23}$$

in terms of the x coordinates of the corresponding points in the physical plane.

A. Single Stripline in a Shielded Box

In this case the conformal mapping provides four points on the real axis (Fig. 3), the abscissas a_1, b_1, a_2, b_2 being determined by relations (21) in terms of L, h, w, and d. Relation (20) gives

$$C/(\epsilon_1+\epsilon_2)=M_{11}/N_{11} \tag{24}$$

where C stands for the total capacitance of the microstrip and M_{11}, N_{11} are the integrals given by relations (19). In the case of a single stripline these integrals can be expressed in terms of complete elliptical integrals of the first

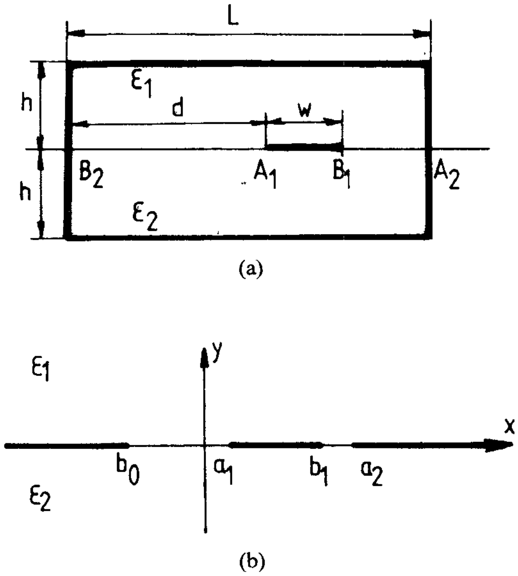

Fig. 3. The geometry of (a) the single stripline and (b) the image in the Z plane.

kind:

$$C/(\epsilon_1+\epsilon_2)=K(s)/K(s'). \qquad (25)$$

Here the modulus s is related to the above-mentioned abscissas by the relation

$$s=\sqrt{\frac{(b_1-a_1)(a_2-b_0)}{(b_1-b_0)(a_2-a_1)}}, \qquad s'=\sqrt{1-s^2}. \qquad (26)$$

The result given in relations (25) and (26) holds for any stripline-like structure shielded in a box.

B. Single Stripline Between Two Parallel Ground Planes

If the length L of the shielding box in Fig. 3(a) becomes infinite, relation (23) will be appropriate and will give

$$b_1=-a_1=\tanh(\pi w/(4h)) \qquad a_2=-b_0=1. \qquad (27)$$

Relation (26) now becomes

$$s=2\sqrt{\tanh(\pi w/(4h))}/[1+\tanh(\pi w/(4h))]. \qquad (28)$$

By using certain relationships between the complete elliptical integrals of the first kind [10], formula (25) can be written as

$$C/(\epsilon_1+\epsilon_2)=2K(k)/K(k'), \qquad k=\tanh(\pi w/(4h)). \qquad (29)$$

Formula (29) was given previously by Cohn [11]. It is used in applications in the form

$$Z_0=\frac{Z_{0v}}{\sqrt{\epsilon_{\text{eff}}}}\frac{K(k')}{K(k)}$$

where $Z_{0v}=\sqrt{\mu_0/\epsilon_0}=376.7\ \Omega$ is the characteristic impedance of free space and $\epsilon_{\text{eff}}=(\epsilon_1+\epsilon_2)/2$ is the effective relative dielectric constant.

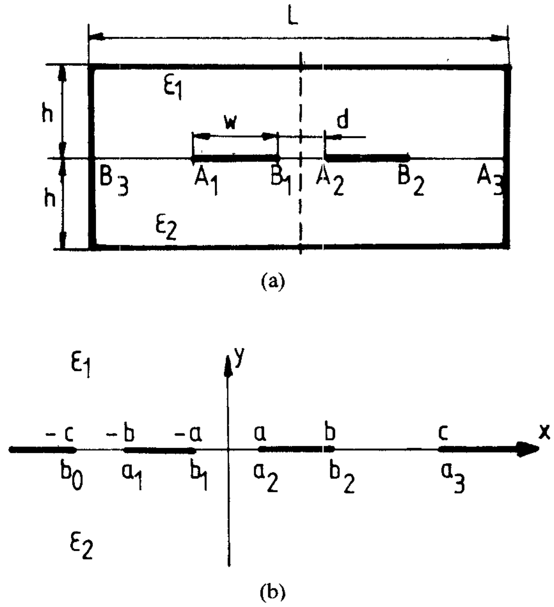

Fig. 4. The geometry of (a) the shielded couple-strip and (b) the image in the Z plane.

C. Two Symmetrical, Coupled Striplines Inside a Shielded Box

If the configuration is symmetrical with respect to a vertical axis (Fig. 4), the capacitances of the system can also be expressed by means of the complete elliptical integrals. By symmetry, the abscissas of the six points of interest are $\pm a, \pm b, \pm c$, where the constants a, b, c are obtained by using the geometrical dimensions L, h, w, d in relations (21) and (22). Relations (18) now give

$$N_{11}(Q_1-Q_2)=(\epsilon_1+\epsilon_2)M_{11}(V_1-V_2)$$
$$N_{21}(Q_1+Q_2)=(\epsilon_1+\epsilon_2)M_{21}(V_1+V_2). \qquad (30)$$

Consider two particular propagation modes: even ($V_1=V_2=V$) and odd ($V_1=-V_2=V$). The capacitances corresponding to these two modes are given by relation (30):

$$C_{\text{even}}/(\epsilon_1+\epsilon_2)=M_{21}/N_{21} \qquad C_{\text{odd}}/(\epsilon_1+\epsilon_2)=M_{11}/N_{11}. \qquad (31)$$

The integrals $M_{11},\cdots,N_{21}$ can again be expressed as complete elliptical integrals of the first kind. We obtain

$$\frac{C_{\text{even}}}{\epsilon_1+\epsilon_2}=\frac{K(p)}{K(p')}, \qquad p=\sqrt{\frac{b^2-a^2}{c^2-a^2}}, \quad p'=\sqrt{1-p^2} \qquad (32)$$

and

$$\frac{C_{\text{odd}}}{\epsilon_1+\epsilon_2}=\frac{K(s)}{K(s')}, \qquad s=\sqrt{\frac{c^2(b^2-a^2)}{b^2(c^2-a^2)}}, \quad s'=\sqrt{1-s^2}. \qquad (33)$$

The two relations (32) and (33) give the capacitance of the symmetrical two-conductor complete shielded coupler.

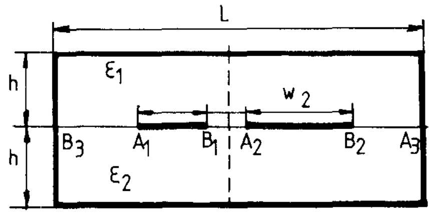

Fig. 5. The nonsymmetrically coupled transmission lines.

D. Two Symmetrical Coupled Striplines Between Two Parallel Ground Planes

When the shielding box has no lateral walls ($L \rightarrow \infty$), formula (23) gives

$$a = \tanh(\pi d/(4h)) \qquad b = \tanh(\pi(d+2w)/(4h)) \qquad c = 1 \tag{34}$$

and therefore relations (32) become

$$\frac{C_{\text{even}}}{\epsilon_1+\epsilon_2} = \frac{2K(k_e)}{K(k_e')},$$

$$k_e = \tanh\left(\frac{\pi w}{4h}\right)\cdot\tanh\left(\frac{\pi}{4}\left(\frac{w+d}{h}\right)\right). \tag{35}$$

We have also

$$\frac{C_{\text{odd}}}{\epsilon_1+\epsilon_2} = 2\frac{K(k_o)}{K(k_o')},$$

$$k_o = \left(\tanh\left(\frac{\pi w}{4h}\right)\right)\Big/\left(\tanh\left(\frac{\pi}{4}\cdot\frac{w+d}{h}\right)\right). \tag{36}$$

Relations (35) and (36) were also obtained by another method by Cohn [12]. They are used in practice for the even and odd characteristic impedances:

$$Z_{0\,\text{even}} = \frac{Z_{0v}}{\sqrt{\epsilon_{\text{eff}}}}\frac{K(k_e')}{4K(k_e)} \qquad Z_{0\,\text{odd}} = \frac{Z_{0v}}{\sqrt{\epsilon_{\text{eff}}}}\frac{K(k_o')}{4K(k_o)} \tag{37}$$

where Z_{0v} and ϵ_{eff} are the same as above.

E. Two Coupled Striplines in a Shielded Box

If the symmetry of the two coupled striplines is given up, no analytical formulas in terms of elliptical functions are available. However, the capacitance matrix can still be expressed by means of the hyperelliptical integrals (19). These can be computed by using numerical methods. To check the formulas (20) we computed the solution for $W_1/W_2 = 1,\cdots,10$ and $d/h = 1,\cdots,10$ (Fig. 5).

In the case where $\epsilon_1 = \epsilon_2$, the values obtained agree with those obtained by Linner [13] by a method working only for homogeneous dielectric media. Some of the results thus obtained were communicated in [14].

F. Multiconductor Structures in a Shielded Box

This is a general case which can be solved by the method developed here. The abscissas a_k, b_k are determined by relations (21) and (22) (in the case of the coupled striplines between ground planes we shall use relation (23)). The

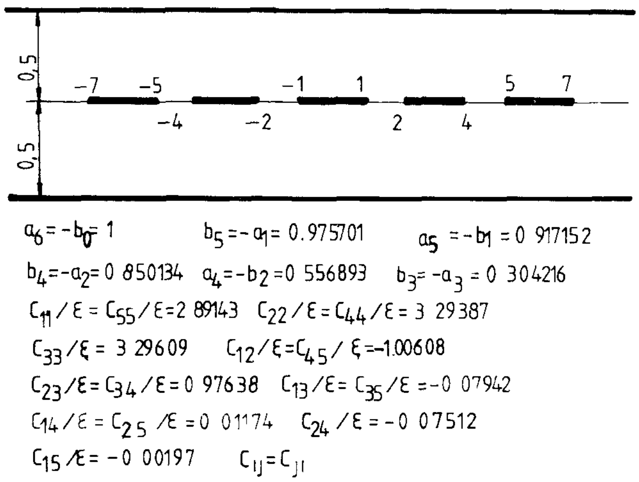

Fig. 6. The geometry of the multiconductor stripline considered in Section IV-F and numerical results.

Maxwellian capacitance matrix is expressed by formula (20), providing an analytical (exact) solution of the problem. Further, the estimation of integrals can be given only by numerical methods.

In order to estimate the accuracy of the method, we considered the case of a multistrip structure with a homogeneous dielectric medium between two parallel ground planes (Fig. 6). This case was studied by Kammler [5]. In Fig. 6 we also give the abscissas a_k, b_k and the numerical results obtained for Maxwell's capacitance matrix.

Notice that the results thus obtained are identical within the first five digits with the numerical results given in [5].

V. Conclusions

The new method offered by this paper is based on conformal mapping and on singular integral equations theory. The use of conformal mapping permits its application to lines having the two dielectric media only if the media have equal heights.

This method provides an analytical expression for Maxwell's capacitance matrix in terms of some hyperelliptic integrals. The influence of the structure geometry on these formulas is expressed only by means of the images of the ends of the conducting strips by conformal mapping.

The method is general, applying to structures with an arbitrary number of conducting strips with arbitrary widths and spacing placed in an inhomogeneous dielectric medium. Moreover, relation (20′) expresses the capacitance of the multistrip system in a stratified dielectric by means of the Maxwell capacitance matrix of the same system placed in free space.

References

[1] R. Garg, "Stripline-like microstrip configuration," *Microwave J.*, vol. 22, no. 4, pp. 103–104, Apr. 1979.
[2] F. Oberhettinger and W. Magnus, *Adwendung der Elliptischen Funktionen in Physik und Technik*. Berlin: Springer, 1949.
[3] H. E. Green "The numerical solution of some important transmission-line problems," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-13, pp. 676–692, Sept. 1965.

[4] E. Yamashita, "Variational method for the analysis of microstrip-like transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 529–535, Aug. 1968.
[5] D. W. Kammler, "Calculation of characteristic admittances and coupling coefficients for strip transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 925–937, Nov. 1968.
[6] T. G. Bryant and J. A. Weiss, "Parameters of microstrip transmission lines and of coupled pairs of microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 1021–1027, Dec. 1968.
[7] G. M. L. Gladwell and S. Coen, "A Chebyshev approximation method for microstrip problems," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, pp. 865–870, Nov. 1975.
[8] D. Homentcovschi, *Complex Variable Functions and Applications in Science and Technique* (in Romanian). Bucharest, 1986.
[9] D. Homentcovschi, A. Manolescu, A. M. Manolescu, and C. Burileanu, "A general approach to analysis of distributive resistive structures," *IEEE Trans. Electron Devices*, vol. ED-25, pp. 787–794, July 1978.
[10] I. M. Ryshik and I. S. Gradstein, *Tables of Series, Products and Integrals*. Berlin: VEB Deutscher Verlag, 1957.
[11] S. B. Cohn, "Characteristic impedance of the shielded-strip transmission line," *IRE Trans. Microwave Theory Tech.*, vol. MTT-2 pp. 52–57, July 1954.
[12] S. B. Cohn, "Shielded coupled-strip transmission line," *IRE Trans. Microwave Theory Tech.*, vol. MTT-3, pp. 29–38, Oct. 1955.
[13] L. J. P. Linner, "A method for the computation of the characteristic immitance matrix of multiconductor striplines with arbitrary widths," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, pp. 930–937, Nov. 1974.
[14] D. Homentcovschi, A. M. Manolescu, and A. Manolescu, "Analysis of multiterminal structures for very high frequencies" (in Romanian) in *Microelectronica*, vol. 15, Ed. Acad. RSR, 1987.
[15] C. Jacob, *Introduction mathematique a la mechanique des fluides*. Paris, 1959.

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